

## IUTAM Symposium on Waves in Fluids: Effects of nonlinearity, Rotation, Stratification and Dissipation

# Ground Effect in Hydrodynamics of a Strip in a Stratified Fluid

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### Abstract

The paper is devoted to the numerical investigation of stratified flow structure and dynamical characteristics of an impermeable obstacle moving at some distance from the rigid horizontal surface. At the simplest cases of a plate moving along underlying plane or in free space the calculation results are compared with the visualizations of exact solution and the Schlieren images of stratified flows in the laboratory experiments. The calculations reveal that accounting for the finestructure effects, i.e. medium stratification and diffusion, which are always present at the natural conditions, can influence essentially upon flow structure, forces and momentum distributions. The obtained results, which have a practical application to ground-effect vehicles and ekranoplans, show the stratification effects may lead to a noticeable increase in drag and decrease in lift of a body moving near the rigid surface.

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### 1. Introduction

Control of aerohydrodynamic properties of objects moving in water or air media is a very important problem nowadays due to a current necessity of efficient energy consumption. One of the most effective approaches in this problem solving is creation of a ground effect, which arises when an aircraft is flying close to the surface of water or the Earth. Moving of a ground-effect vehicle, an airfoil boat or an ekranoplan at short distances from the surface generates an additional lift force, the full effect being observed at cruising altitudes not greater than length of the aircraft's wing chord. The reflected approach flow reaches the surface and returns back, thus creating a region of compressed flow or some kind of a dynamic air cushion.

Theoretical evaluations and numerical calculations of dynamical characteristics of moving bodies are conducted, as a rule, in the homogeneous fluid approximation when effects of real medium properties and external

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dynamical factors are neglected. But at the natural conditions fluid media are exposed to numerous internal and external factors including dissolved and suspended matters, gas bubbles, temperature, medium compressibility, humidity, different external non-stationary factors etc, which can drastically influence upon fluid density and its gradient. Even a small medium non-homogeneity may change character of fluid flows, structure of vortices, forces and momentum distributions.

Accounting for the finestructure effects introduces usually not big corrections to a total value of drag and lift forces but they may be comparable with a profit obtained by an application of flow control methods. It is well-known that aerohydrodynamical drag reduction even up to several percents may lead at great scales to a significant economy of energy resources. The additional factors are related with existence of internal waves (IW).

IW are ubiquitous in the Earth's oceans and atmosphere, where they are generated by flow over ocean-floor topography and sea-surface winds [1 – 3]. IW transfer energy and momentum to large distances and form turbulent spots in the case of their breaking, which intensifies the substances transport in the ocean and effects the flights safety in the atmosphere. Dissipation of these IW impacts the Earth's climate by influencing large-scale ocean circulation, affecting plankton distribution, and perhaps even shaping the continental slopes. From an engineering perspective, internal waves affect the performance of underwater technology, such as acoustic communication, submersible vehicles and marine cabling. A detailed understanding of all aspects of IW generation and evolution is therefore both profoundly and practically important.

Numerous theoretical and experimental works are devoted to the investigation of the attached (lee) IW formed by flowing around the obstacles [2, 4 – 8]. In view of the inconsistency of equations and boundary conditions, the numerical analyses of the wave fields are, as a rule, performed in the linear approximation and the actual body is replaced by a number of hydrodynamic sources and sinks in the approximations of viscous and ideal exponentially stratified fluids. A method for construction of exact solutions of fundamental system for non-homogeneous fluids in the linear approximation was proposed in [6]. This analytical study was applied to the problem on generation of IW by a moving strip in the underlying plane approximation. But the application of the artificial boundary conditions makes it difficult to extrapolate the data to the environmental conditions and practically valuable range of parameters [2]. Numerical methods allowed overcoming these difficulties and considering the problem in full formulation for a moving obstacle in free space [7]. The both approaches demonstrate similar flow patterns including two groups of IW with their structure depending essentially on trajectory slope angle to the horizon and compact non-wave singularities near the plate's edges [8].

The present paper is devoted to investigation of stratified flow finestructure and dynamical characteristics of an impermeable obstacle moving at some distance from the rigid horizontal surface on the basis of numerical approaches applied to the non-linear equations of continuously stratified fluid motion. At the simplest cases of a plate moving along underlying plane or in free space the calculation results are compared with the visualizations of exact solution and the Schlieren images of stratified flows in the laboratory experiments.

## Nomenclature

$(x, y, z)$	laboratory coordinate system
$\mathbf{v}$	velocity vector
$P$	pressure perturbation
$S$	total salinity
$s$	salinity perturbation
$\nu$	kinematic viscosity
$\kappa_S$	molecular diffusivity

$\mathbf{g}$	gravitational acceleration
$\rho$	fluid density
$\Lambda$	stratification scale
$N_b$	buoyancy frequency
$L$	plate length
$U$	velocity of plate movement
$\mathbf{v}_{dif}, S_{dif}$	initial fields of diffusion induced steady flow

## 2. Problem statement and numerical solution

For numerical analysis of the non-stationary problem on IW generation by a plate moving near the rigid horizontal surface we consider the fundamental set of equations for continuously stratified fluid. The governing system includes the non-stationary Navier – Stokes equation accounting for the gravity in the Boussinesq approximation, the continuity and the diffusion equations and the closing linearized state equation

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}(\nabla \cdot \mathbf{v}) &= -\frac{1}{\rho_0} \nabla P + \nu \Delta \mathbf{v} - s \mathbf{g} \\ \text{div } \mathbf{v} &= 0; \quad \frac{\partial s}{\partial t} + \mathbf{v} \cdot \nabla s = \kappa_s \Delta s + \frac{\mathbf{v}_z}{\Lambda}; \quad \rho = \rho_0 \left( 1 - \frac{z}{\Lambda} + s \right) \end{aligned} \quad (1)$$

Initial values of the basic physical parameters are nonzero and they are determined based on numerical solution of the problem on diffusion induced flow around an impermeable plate immersed in a quiescent continuously stratified fluid. Such flows are characterized by rather a complex system of fluid motions, which compensate breaking of diffusion flux on an impermeable surface, including main along-surface jet flows and secondary circulating cells with a complex multilevel structure [9]. Then the plate starts to move instantly with uniform velocity  $U$  along the horizontal axis. The physically valid initial and boundary conditions consist of given distributions of the fields at the initial moment of time, no-slip and no-flux boundary conditions for velocity and salinity on the plate surface and attenuation of all perturbations at infinity:

$$\begin{aligned} \mathbf{v}|_{t \leq 0} &= \mathbf{v}_{dif}(x, z), \quad s|_{t \leq 0} = s_{dif}(x, z) \\ \mathbf{v}_x|_{\Sigma} &= U, \quad \mathbf{v}_z|_{\Sigma} = 0, \quad \mathbf{v}_x|_{sf} = \mathbf{v}_z|_{sf} = 0 \\ \left[ \frac{\partial s}{\partial \mathbf{n}} \right]_{\Sigma, sf} &= -\frac{1}{\Lambda} \frac{\partial z}{\partial \mathbf{n}} + \left[ \frac{\partial s}{\partial \mathbf{n}} \right]_{\Sigma, sf} = 0, \quad \mathbf{v}, s|_{x, z \rightarrow \infty} = 0 \end{aligned} \quad (2)$$

The posed problem (1), (2) is solved numerically using the finite difference method of second order accuracy with splitting for physical parameters on spaced “chessboard” grid for spatial derivatives.

With the purpose of verification of this numerical method and outlook to consider more complicated geometries we used, as well, another approach based on consideration of external stratified fluid flow around a motionless plate. A constant value of free stream velocity, zero values for velocity components and total salinity flux and moving wall conditions, as well, are set at the inlet, the plate’s surface and the underlying plane, respectively. At the outlet and the upper boundary of the calculation domain “soft” boundary conditions are used, i.e. zero values for surface-normal gradients of velocity, pressure and salinity perturbation. The posed problem with the men-

tioned boundary conditions is solved using finite volume method, which nowadays is the most popular numerical approach. The method is realized in the frame of independently developed solvers of the OpenFOAM package, which is a free and open source software for the development of customized numerical solvers for computational fluid dynamics. The data obtained by the both approaches are compared with each other for numerical errors controlling and checking on validity of physical results.

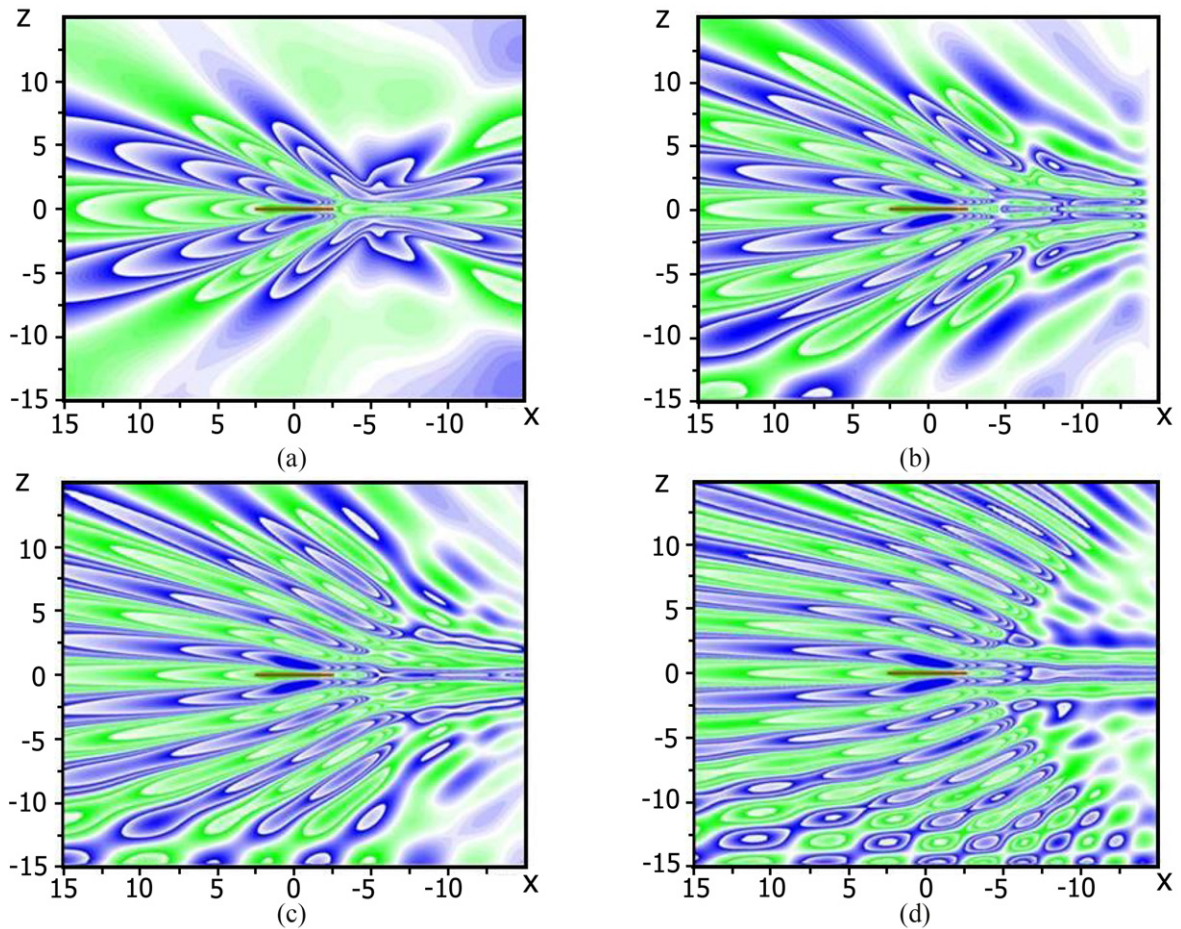


Fig. 1. Evolution through time of horizontal velocity component field generated by the horizontally moving plate at some distance from the surface in a continuously stratified fluid. (a)  $\tau = 2$ . (b)  $\tau = 4$ . (c)  $\tau = 6$ . (d)  $\tau = 12$ .  $U = 0.47$  cm/s,  $N = 1.26$  s<sup>-1</sup>,  $L = 2.5$  cm,  $h = 15$  cm.

The calculations of the problems were run in parallel regime on SRCC MSU and JSCC RAS supercomputer facilities using the public domain openMPI implementation of the standard message passing interface (MPI). The method of parallel computing is known as domain decomposition, in which the geometry and associated fields are broken into pieces and allocated to separate processors for solution.



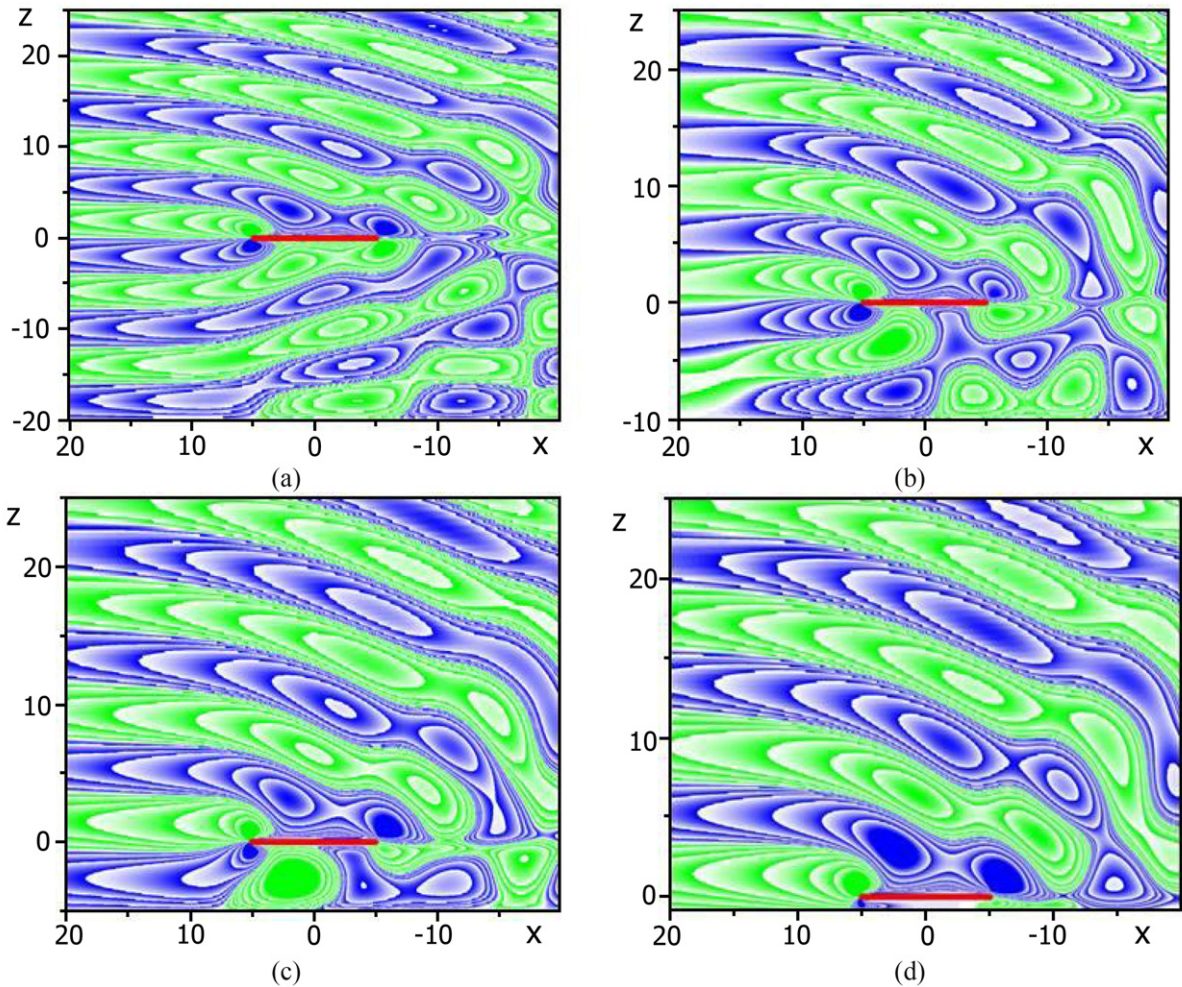


Fig. 2. Steady flow patterns of vertical velocity component field generated by the horizontally moving plate at different distances from the surface. (a)  $h = 20$  cm. (b)  $h = 10$  cm. (c)  $h = 5$  cm. (d)  $h = 0.5$  cm.  $U = 1$  cm/s,  $N = 0.83$  s $^{-1}$ ,  $L = 10$  cm,  $\tau = 20$ .

The numerical data were post-processed using the graphical interface, ParaView, and the program package, Origin, with the application of a high-resolution method of polychromatic contour map construction. Such an approach allows a programmer to extract both qualitative and quantitative information from the constructed fields for different physical parameters. The colour information on a field structure between neighbouring isolines is presented in the form of a continuous gradation spectrum ranging from white to green (light-gray) colours for positive values of a chosen physical variable and from blue (dark-gray) to white colours for negative ones [2].

### 3. Computational results and discussion

Let us first consider process of evolution through time of IW patterns around a plate moving at some distance from the rigid surface,  $h = 15$  cm, which are presented in Fig.1. The initial stage of the flow evolution around a horizontally moving plate is characterized by an origin of perturbations near the plate's edges, which are the only sources of IW since vertical displacements of fluid particles are absent along the whole length of the plate.

At this stage the perturbations do not reach the surface and the flow pattern keeps a symmetric form relative the horizon as in the case of a moving plate in free space (Fig.1(a)). Further flow evolution represents the initial perturbations' expansion to far flow field and the formation of advanced and attached IW in front of and in the wake of the plate (Fig.1(b),(c)). At this stage a noticeable evidence of reflected waves appears in the form of distortions in the patterns of advanced and attached waves fields both in the flow region under the plate and, to much less extent, at the upper half-space (Fig.1(d)). At the stationary stage when flow patterns stop changing in time the phase surfaces of the advanced IW are stretched along the horizon near the trajectory of movement gradually decaying with increasing distance from the plate.

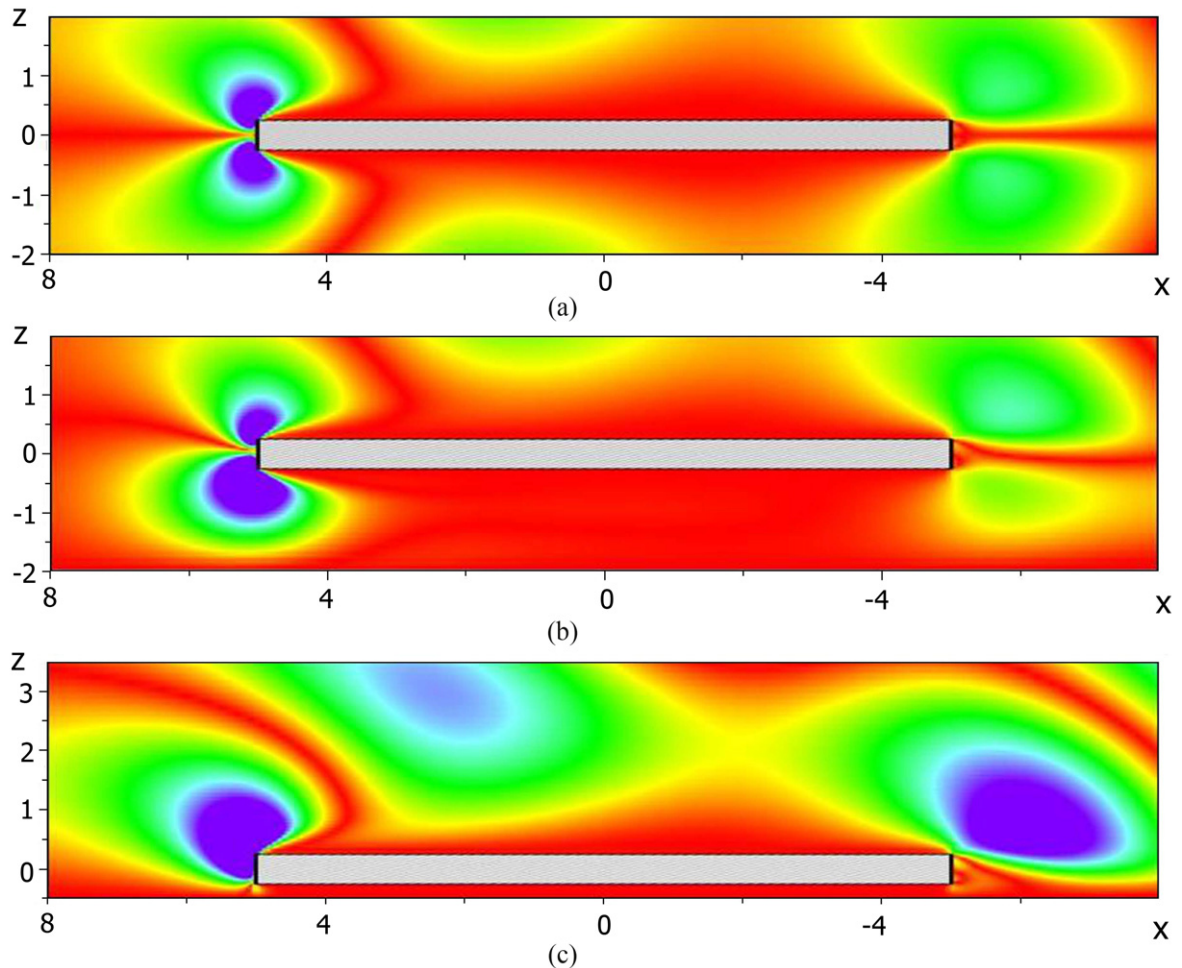


Fig. 3. Pattern of module of vertical velocity component field generated by the horizontally moving plate at different distances from the surface. (a)  $h = 20$  cm. (b)  $h = 2$  cm. (c)  $h = 0.5$  cm.  $U = 1$  cm/s,  $N = 0.83$  s $^{-1}$ ,  $L = 2.5$  cm,  $\tau = 20$ .

The steady patterns of vertical component of velocity field are presented in Fig.2 for different distances from the plate to the rigid surface. A steady pattern of stratified flow turns to be essentially asymmetric relative to the plate's horizon especially in the wake of the plate even when it is located at high altitudes from the underlying surface,  $h = 20$  cm (Fig.1(a)). With shortening the distance from the plate to the surface flow pattern in the region under the plate takes very complicated form due to intensive processes of IW interactions and reflections in the limited space (Fig.1(b)). Structure of the flow region under the plate strongly depends on the relation of dis-

tance to the surface and length of the attached IW,  $h/\lambda = h/UT_b$ . At distances,  $h < 5$  cm, the stratified flow pattern at the upper half-space turns to be qualitatively very similar to that obtained analytically in the linear approximation for a moving plate along underlying plane [2] (Fig.1(c),(d)).

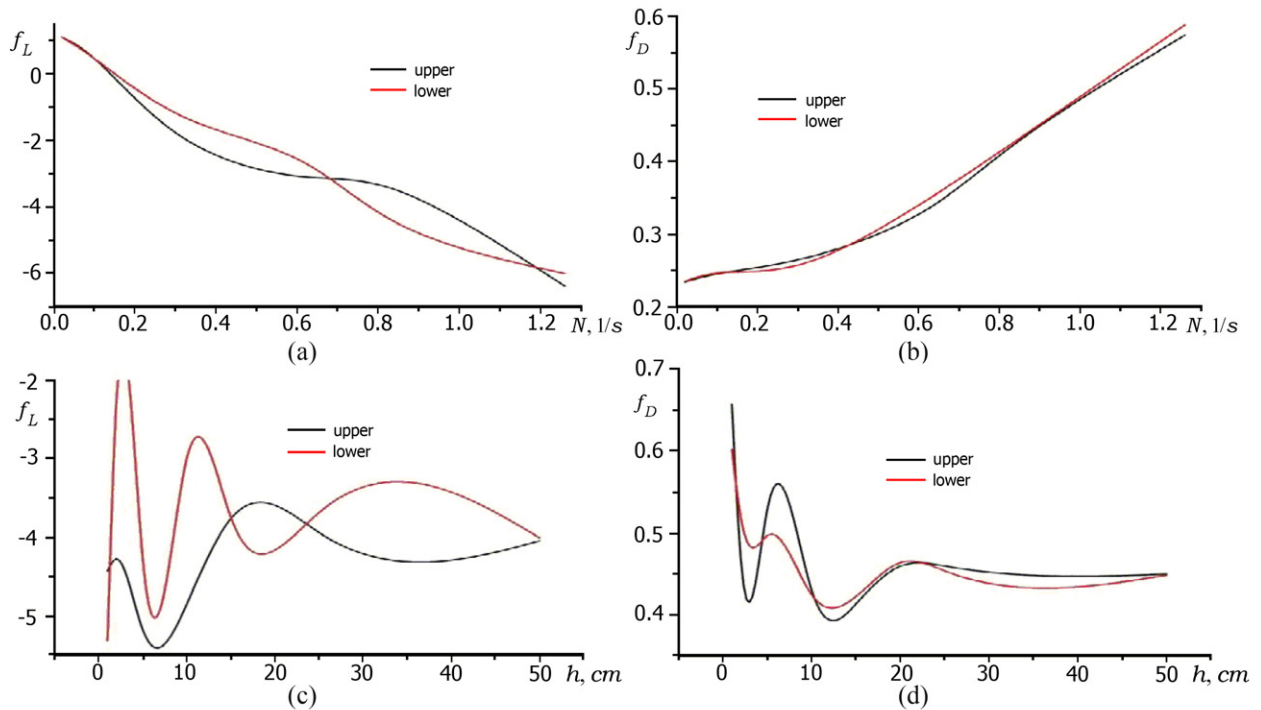


Fig. 4. Dynamical characteristics of the plate vs buoyancy frequency and distance to the surface. (a,c) lift force. (b,d) drag force.  $U = 1$  cm/s,  $L = 10$  cm,  $\tau = 20$ .

Change in distance between the plate and the surface effects essentially upon structure of regional singularities near the plate's sharp edges. At rather great values of  $h > 20$  cm the pattern of module of vertical velocity component field, which is shown in Fig.3, is almost symmetric relative to the plate's horizon. At the same time intensity of the regional singularities turns to be much higher at the front edge of the plate than it does at the trailing one (Fig.3(a)). With shortening the distance to the surface some expansion of the region with maximal perturbations is observed near the front lower edge of the plate (Fig.3(b)) but at  $h < 2$  cm the intensive singularities are located mainly at the plate's upper edges (Fig.3(c)). At the extreme case of the plate moving very closely to the underlying surface (Fig.3(d)) intensity of the regional singularities is almost the same in absolute values at both the front and the back edges of the plate that is well verified by the results of numerical visualization of exact solution [2].

Calculations of forces acting upon the horizontal plate demonstrate that effects of stratification and diffusion influence essentially on the dynamical characteristics of the moving object. With an increase in value of fluid stratification the total lift force decreases almost in direct proportion to buoyancy frequency but at the same time the drag force grows (Fig.4(a,b)). At comparatively great values of stratification the integral forces acting upon the upper and the lower sides of the plate differ significantly in absolute values while at small  $N$  they are practically indistinguishable. Dependencies of integral characteristics on distance to the surface turn to be essentially non-monotone, behaviour pattern of the curves in Fig.4(c,d) is determined by a sign of attached IW's phase on the underlying surface.



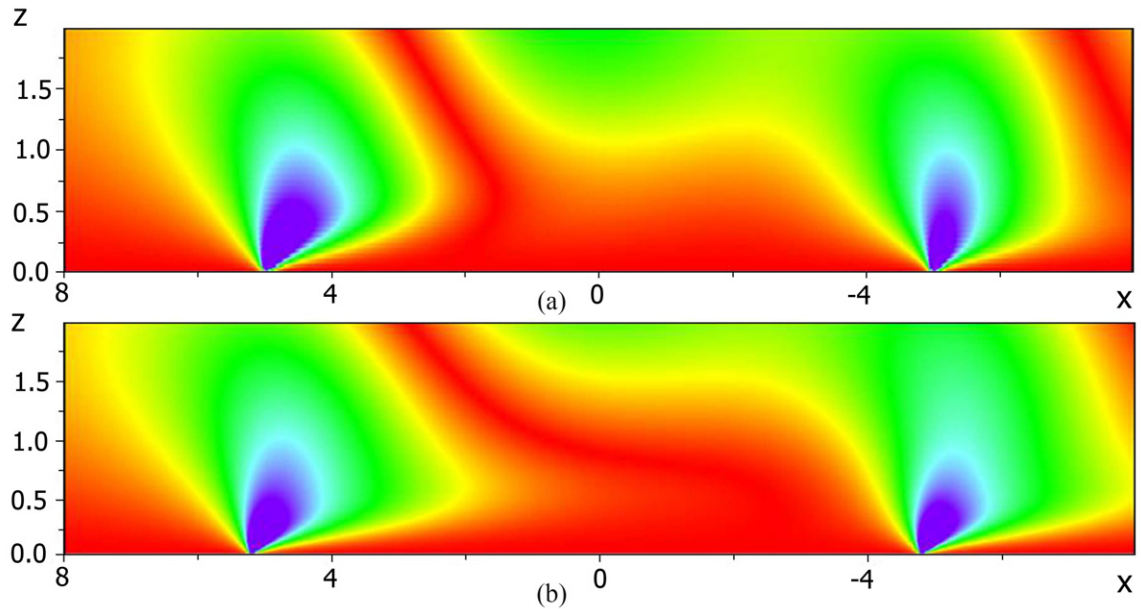


Fig. 5. Stratified flow field near the plate moving along the underlying plane. (a) calculated flow pattern. (b) visualization of exact solution.  $U = 1 \text{ cm/s}$ ,  $N = 0.83 \text{ s}^{-1}$ ,  $L = 10 \text{ cm}$ ,  $\tau = 20$ .

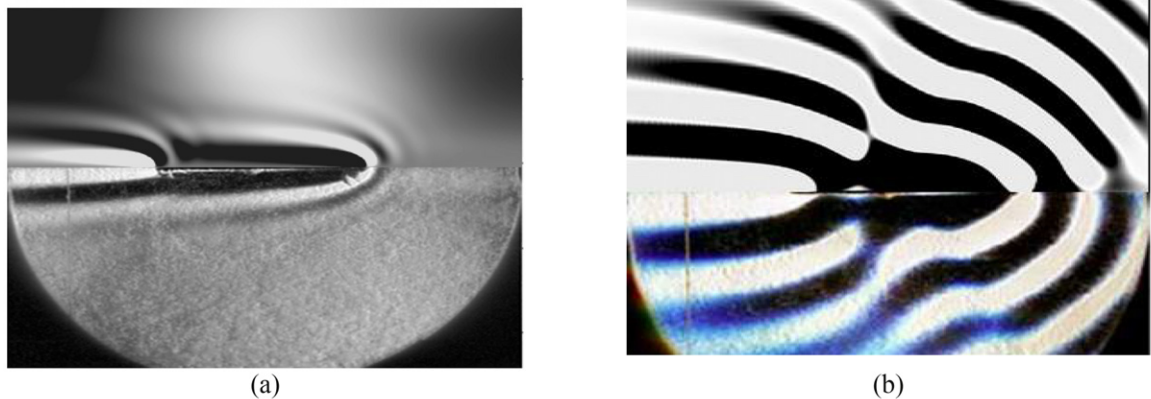


Fig. 6. Synthetic flow pattern of vertical velocity component field (the upper part of the images) over schlieren image of stratified flow generated by uniformly moving horizontal plate. (a)  $U = 0.32 \text{ cm/s}$ . (b)  $U = 0.47 \text{ cm/s}$ .  $L = 7.5 \text{ cm}$ ,  $N = 0.83 \text{ s}^{-1}$ .

The comparison of numerical, analytical and laboratory data are presented in Fig.5 and Fig.6. The stratified flow patterns near the plate moving along the underlying surface turn to be very similar for the direct numerical modeling and the visualization of exact solution. Some differences between the results based on the two approaches are observed in the structure of regional singularities at the trailing edge of the plate and in the intensities of singularities due to the accounting for the effects of non-linearity and diffusion only in the numerical modeling. The calculated fields of IW are in a good agreement with the high-resolution schlieren images of stratified flows around the horizontal plate uniformly moving in free space [4, 10] (Fig. 6). Observed and calculated shapes of IW crests and troughs are compatible even in fine details.



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